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Costin-Ionuţ Dobrotă and Alexandru Stancu

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What does a first-order reversal curve diagram really mean? A study case: Array of ferromagnetic nanowires

Costin-Ionuț Dobrotă and Alexandru Stancu
Faculty of Physics, “Alexandru Ioan Cuza” University of Iasi, Iasi 700506, Romania

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The magnetic characterization technique of hysteretic materials based on the measurement of the first-order reversal curves (FORC) is one of the most appealing methods recently introduced in hundreds of new laboratories, but due to the complexity of the FORC data analysis, it is not always properly used. This method originated in identification procedures for the classical Preisach model and consequently often the FORC distribution is interpreted as a slightly distorted Preisach distribution. In this paper, we discuss this idea from two points of view derived from the basic assumptions used in the Preisach model. One is that the interaction field is equivalent with a shift of the rectangular hysteron along the applied field axis without changing the intrinsic coercivity. The other is the direct use of switching fields as coordinates, in fact, the ones defining the Preisach plane. We discuss the compatibility between the experimental FORC distribution and the Preisach model developed on the interaction field hypothesis. As a “toy model,” we are using a system of ferromagnetic nanowires, explaining from the physical point of view the complex FORC diagrams as they are obtained in experiments. This explanation gives a fundament for the correct interpretation of the FORC diagram in order to get “Preisach type” information about the system, mainly about the distributions of coercive and interaction fields within the sample. These results are relevant for many ferromagnetic systems and give a valuable guide for understanding the FORC technique and its fundamental link with the Preisach model. © 2013 American Institute of Physics. [http://dx.doi.org/10.1063/1.4789613]

I. INTRODUCTION

The number of laboratories and groups, using in their research various hysteretic processes the experimental technique based on the measurement of the first-order reversal curves (FORC), is continuously increasing due to the confirmed method’s exceptional sensitivity and to the multitude of information provided by this technique compared to, for example, the use of the data from the major hysteresis loop (MHL). This is basically due to the fact that the FORC method is not introducing, from the experimental point of view, a qualitatively new type of experiments. Any laboratory that can measure MHLs can extend the measurement for the FORC technique with rather minor adjustments. The significant increase of the number of data required by this method is balanced by supplementary information given by the FORC distribution that can be calculated from the FORC data. However, the use of this method is not yet generalized due to inherent numerical and theoretical complexity in calculating the relevant diagram and getting correct understanding of the results.

For any hysteretic process analyzed with this method, a fundamental hysteresis brick has to be properly defined (usually it is named hysteron, characterized by the switching fields between the two stable equilibrium states called “up” and “down”1,2). The hysteron can also be defined using two equivalent parameters: (i) the coercivity describing the width of the hysteresis loop, and (ii) the interaction field which is related to the shift of the hysteron hysteresis along the field axis. The non-interacting case is represented by the symmetric hysterons which could be, for example, the hysteresis loop of an isolated ferromagnetic entity (like a single domain particle with a hysteresis loop calculated with the Stoner-Wohlfarth model of coherent rotations3). When a FORC experimental set of data is measured, the goal is to calculate a distribution of coercive and interaction fields of the fundamental hysteretic entities (hysteron that can be associated directly with ferromagnetic particles and/or even with ferromagnetic domains). However, structured ferromagnetic samples show complex FORC diagrams4-17 regarded merely as fingerprints of systems without a clear physical understanding.

In this paper, we are giving a broad discussion of the reality behind the FORC diagram for arrays of ferromagnetic nanowires in an axial applied field. We have chosen this system because the component magnetic entities, the ferromagnetic wires, have a virtually rectangular hysteresis loop18,19 when the wires are isolated. Considering identical and parallel wires in a perfect geometrical network, the system seems to be simple enough to provide a good test for a model using interacting rectangular hysterons like in the Preisach model1,2. We have focused our analysis on the information provided by the experimental FORC technique and on the physical reality described by the FORC distribution. In Sec. II, we discuss the notions used in this study (hysteron, Preisach distribution, switching events, etc.). In Sec. III, a physical Ising-type model20 for the wire networks is presented. Sections IV and V are dedicated to the discussion of the FORC results in two different approaches used in Preisach theory: (i) interactions are
shifting the hysterons along the field axis without changing their coercivity, and (ii) interactions are producing changes in the values of the switching fields, with no other constraints.

In Sec. VI, the results are discussed and compared and we suggest a simplified method to properly use the FORC data for networks of nanowires in order to obtain valuable physical data about the system: the intrinsic coercivity distribution, the inter-wire interaction intensity, and the real switching sequence of the wires.

II. HYSTERONS IN THE PREISACH DISTRIBUTION: COERCIVITY, INTERACTIONS, AND SWITCHING FIELDS

Before we start the discussion concerning the physical significance of the FORC distribution, we introduce this concept in slightly more detail. A FORC is an experimental curve completely confined inside the MHL. In fact, a FORC starts on one of the two branches of the major loop. If one discusses the FORC set that originates on the descending branch of the MHL, the sequence of applied fields before the start of a FORC measurement is beginning with a field sufficient to saturate the sample on the positive direction. Then the field is decreased down to a field called the reversal field, \( H_r \). One FORC (identified by \( H_r \)) is measured between \( H_r \) and the field sufficient to saturate the sample again. In a typical FORC set, we can have up to 100 such curves that cover the entire surface of the major loop [see Fig. 1(a)]. This type of experiment was originally designed by Mayergoyz as an identification technique for the classical Preisach model (CPM). The idea was that a point on a FORC is a function of two fields, \( H \) and \( H_r \), and the Preisach distribution can be obtained from these data by numerical second order mixed derivative

\[
p(H, H_r) = -\frac{1}{2} \frac{\partial^2 m_{\text{FORC}}(H, H_r)}{\partial H \partial H_r}.
\]

However, Mayergoyz based his mathematical demonstration (that Eq. (1) is giving the Preisach distribution) on two necessary and sufficient conditions/properties: (i) the wiping-out property (minor loops should perfectly close after one cycle); and (ii) congruency of minor loops measured inside the MHL between the same field limits regardless the history of the field applied before the measurement of these minor loops. The minor loops are considered congruent if they could be superimposed using only translations along the magnetic moment axis (ordinate). Unfortunately, most of the experiments performed on ferromagnetic samples of various origins have shown disagreements with one or both conditions.

In 1999, Pike and collaborators gave a new vision on the FORC identification technique. Essentially, they said that the FORC procedure can be considered as an independent experimental technique and should be used “as such.” As a result, the experimentalists will have a FORC distribution, calculated also with Eq. (1), but the result will not be the Preisach distribution but the FORC distribution, \( \rho(H, H_r) \)

\[
\rho(H, H_r) = -\frac{1}{2} \frac{\partial^2 m_{\text{FORC}}(H, H_r)}{\partial H \partial H_r}.
\]

Although Pike underline that the FORC distribution is not the Preisach distribution, most of the users were inclined to imagine the FORC distribution as a slightly distorted Preisach distribution of the sample. Unfortunately, this assumption has not been seriously tested in all cases and in this paper we show that some of the conclusions extracted from this hypothesis could be simply incorrect.

To perform this analysis, we have chosen a physical system that has an intrinsic simplicity and can be related to a...
the straightforward manner to an ideal Preisach system of rectangular hysterons with magnetostatic interactions. Experimental and micromagnetic studies, have shown that the fundamental brick of a magnetic nanowire has a rectangular hysteresis loop, similar to a Preisach hysterons. The interactions between wires can be calculated as the field created by all the other wires in the center of the studied wire. Systematic studies have shown that in the case of large assemblies of long wires the field along the wire is almost constant. Everything is in place like for an ideal physical template for a Preisach-type model. However, since the first use of the FORC technique on nanowire systems, many experiments have shown that these systems are not compatible with the CPM. Experiments testing wiping-out and congruency properties apparently have shown that they are obeyed. However, later experiments have shown that congruency is in fact not obeyed and the experiment reported in Ref. seemed to have proven the congruency property could not be properly documented.

The fact that most of the ferromagnetic systems do not obey to the congruency property was, in a way, a common knowledge in the Preisach community (for example Brown’s comments). Since the early attempts to measure the Preisach distribution, many years before the FORC technique was designed, it has been observed that something was not right with the Preisach distribution as it is comprehended in the CPM.

Della Torre has developed a modified version of the Preisach model, known as the moving Preisach model (MPM), in which the Preisach distribution is moving along the interaction field axis as a function of the total magnetic moment of the sample. Later, this condition was further relaxed by Martha Pardavi-Horvath and Della Torre in the variable variance Preisach model (VVPM) where the variance of the interaction field distribution is also considered to be dependent on the magnetic state of the sample (the FORC diagrams obtained with the MPM were systematically analyzed much later).

Nevertheless, when experimental FORC diagrams measured on magnetic nanowires are studied, some unexpected features can be observed. The typical ferromagnetic nanowire array FORC diagram has two characteristic regions. One takes the form of a narrow distribution of coercive fields with a wide distribution of interaction fields, indicating strong magnetostatic interactions. We shall refer to this specific feature as to interaction field distribution (IFD). The second distribution is less prominent than the first one and has a large dispersion of coercive fields, apparently describing non-interacting nanowires. We shall refer to this second distribution as to coercive field distribution (CFD). We mention that this distribution can be observed in all experimental FORC diagrams of nanowire arrays, including complex structures like segmented (barcode) nanowires consisting of alternating magnetic to non-magnetic segments or complex wires with a single modulated diameter structure. A number of hypotheses have been launched in order to explain these specific FORC diagrams, especially about the occurrence of the CFD. Geometrical characteristics of the nanowire arrays have been analyzed, assuming that a length distribution of the wires or non-uniformities from the initial growth parts could cause the CFD. It was found, however, that after removing these inhomogeneities by polishing the nanowire array, the CFD persists in the same region of the FORC diagram. Analyzing Ni nanowires grown in different diameter templates and wires with single modulated diameter, the CFD appears in all FORC distributions but it is more prominent in the case of small diameter nanowires. On the other hand, it was considered that structural properties of crystalline bi-phase Co nanowires could be the reason of the two distributions, assigning the CFD to the fcc phase which switches after the hcp phase completely reverses. Nevertheless, this is just a particular case as long as the CFD is visible in any crystalline or amorphous nanowire arrays.

Unfortunately, none of these hypotheses entirely clarified the physical source of the IFD/CFD FORC diagram structure in terms of the Preisach models. Since the first FORC diagram was obtained for Ni nanowire arrays by Spinu et al., the large extension along the interaction field axis has been observed and it was emphasized that the IFD does not represent the Preisach distribution in a certain state of the sample. Most of the published articles provide as physical source of the IFD the effect of demagnetizing-type mean field overlapped on the statistical distribution of interactions, in terms of the MPM, admitting that the MPM is not a complete description of a nanowire array. As a quantitative result, the half-width of the IFD extension along the interaction field axis was identified as the maximum interaction field in the array in several publications. The coercive field distribution revealed from the entire FORC diagram was identified as the real coercivity distribution of the wires from the system. The coercive field distribution along the interaction field axis was sometimes named an “artifact” and it is not even discussed in other papers. A link between the existence of the CFD and the presence of a “reversal field memory” effect was observed in Ref. 14 and theoretical descriptions of larger coercive fields were related to the positive mean interaction field in Refs. 34 and 35. An increase of the coercivities was presented by Pike in a model for the “wishbone” structure of the FORC diagram characteristic for bit patterned media (BPM) samples. A physical explanation for the large coercivity branch of the FORC diagram was formulated in terms of switching fields.

The Preisach model has a fundamental problem to explain the CFD region. This region suggests the existence of wires with considerably larger coercivity than the isolated wires. A study of the CFD should start from the definition of the elementary hysteron in the Preisach model. There are two ways to find the “position” of the magnetic moment of one hysteron in the Preisach plane: (i) using the interaction field and the intrinsic coercivity; and (ii) using directly the switching fields as coordinates: the negative switching field from “up” to “down,” \( H_f \) (equivalent with \( H_r \) in a FORC experiment), and the positive switching field from “down” to “up,” \( H_s \) (equivalent with \( H \) in a FORC experiment), respectively.
The FORC diagram is the contour plot representation of the distribution function (2), as we present in Fig. 1(b) using \((H, H_c)\) coordinates, or using the coercive field axis

\[
H_c = \frac{(H_z - H_0)}{2}
\]

and the interaction field axis

\[
H_u = \frac{(H_z + H_0)}{2} = -H_i,
\]

as in the rotated plane from Fig. 1(c). In fact, \(H_u\) is the “bias field” meaning the shift of the rectangular hysteresis loop as considered in the CPM (see Fig. 1), but it is usually named “interaction field,” with the comment that the real interaction field is \(H_i = -H_u\).

### III. ISING-TYPE MODEL FOR THE FORC DIAGRAMS

To give a physical interpretation of the nanowire arrays FORC diagrams, we propose a very simple model that accounts for state dependent interactions between wires.\(^\text{14,20}\) We consider a rectangular nanowire array of \(40 \times 40 (N = 1600)\) cylindrical wires with the same length \((L = 6 \mu m)\) and radius \((R = 40 \text{ nm})\) perfectly ordered in a 2-D square grid with the constant \(a\) (interwire distance). Following Ref. 19, Nickel nanowires with the length larger than \(1 \mu m\) may be approximated as magnetic dipoles (macrospins) in an axial applied field. In this case, inhomogeneous states of a nanowire are just transient states, and at a given applied field, all nanowires are homogeneously magnetized, not necessarily in the same direction.

The magnetostatic interaction field, in axial direction \(z\), created by a macrospin, at distance \(x\) on the mediator of the dipole is given by

\[
H_z = \frac{\pi R^2 L M_s}{\left(x^2 + L^2/4\right)^{3/2}},
\]

where \(M_s = 485 \text{ emu/cm}^3\) is the saturation magnetization for Nickel used in the next simulations. The field created in the center of each wire, \(H_{k_{\text{eff}}} (k = 1, \ldots, N)\) is evaluated as the sum of interaction fields created by all the other wires. When the array is magnetically saturated in an axial applied field, e.g., in the positive saturation state, the interaction field in each wire is opposed to the applied field and it has a demagnetizing effect. In all simulations, we have fixed the same geometrical parameters of the wires, and we have used different interwire distances in order to obtain interaction fields with various magnitudes. Real nanowire arrays show inevitably slight non-uniformities of the geometrical characteristics of the individual wires, small deviations from parallelism and possible structural defects or imperfections that can slightly modify the individual switching fields,\(^\text{36}\) so, to account for that, a small dispersion of coercivities was considered in simulations for analyzed arrays. Our assumption that all wires are geometrically identical is an approximation for saving computational time, but a slight dispersion of coercive fields could be attributed to defects and imperfections. A normal distribution of the critical fields was chosen, with the average field \(H_{c0} = 150 \text{ Oe}\) and the standard deviation \(H_{c \sigma} = 20 \text{ Oe}\), with \(H_{ck}\) the individual intrinsic coercive fields.

Due to the shape anisotropy, magnetic moments of the nanowires, \(m_k\), are oriented parallel to the wire axis. Werndorfer and coworkers\(^\text{18}\) have shown experimentally that the magnetization reversal in an isolated Ni nanowire is a nucleation-propagation process initiated at the ends of the wire, with a switching time of about \(10^{-8} \text{ s}\). Micromagnetic simulations performed by Hertel\(^\text{19}\) have confirmed the experimentally observed switching process for a small set of magnetostatically coupled Ni nanowires, underlying the influence of magnetostatic interactions on the switching fields. Since the wires are switching very fast in contrast to typical VSM measurement time (usually 1 s), we can approximate this process with an instantaneous reversal of the macrospin, when the coercive field is reached.

To calculate the descendant branch of the MHL, the applied field, \(H_{\text{app}}\), was initially set at a value ensuring the positive saturation of the array, when the magnetic moments of the nanowires are \(m_k = \pi R^2 L M_s\). Then the applied field is successively diminished with a small field step and nanowires are randomly checked if they switch or not. If \(m_k > 0\) and the effective field \(H_{k_{\text{eff}}} = H_{\text{app}} - H_{ck}\), then the magnetic moment becomes \(m_k = -\pi R^2 L M_s\), and after each switching event, the interaction fields are updated. In this way, we can track the evolution of the interaction field that each particle is subjected to, during the magnetization process. We have actually defined a zero Kelvin (all the thermal effects were neglected) Ising-type model in which intrinsic anisotropies were taken into account. In Ref. 20, we named this type of model Ising-Preisach to stress the links with

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**Figure 2.** Modeled FORC diagrams for different strengths of the interaction fields set up by interwire distance \((a)\), for \(40 \times 40\) nanowire arrays with \(R = 40 \text{ nm}, L = 6 \mu m, M_s = 485 \text{ emu/cm}^3\), assuming normal distributed coercive fields with the average \(H_{c0} = 150 \text{ Oe}\) and the standard deviation \(H_{c \sigma} = 20 \text{ Oe}\).
these two models. In this case, we have implemented a zero Kelvin Ising-Preisach model (0K-IPM) for nanowire arrays.

In Fig. 2, we present simulated FORC diagrams for interacting nanowire arrays, for different values of the characteristic interwire distance (rectangular network constant). In the case of weak or moderate interactions (for $a = 550$ nm and $a = 450$ nm), we observe FORC diagrams with a “wishbone” shape, which illustrate the presence of the mean field interactions of demagnetizing type. The same kind of diagram is experimentally observed in the case of perpendicular recording media, like patterned media and 2D granular-type layers. For networks with stronger interactions (small $a$), we observe more and more clearly the two distributions mentioned before, IFD and CFD in agreement with experiments.

**IV. INTERACTION-COERCIVE FIELDS BASED PREISACH MODEL**

In order to develop a consistent physical image of the network of nanowires FORC diagram based on the Preisach model, we start from the following remark: if the Preisach hysterons have the same coercivity as in non-interacting state (isolated wires) and the interactions are variable shifting the loops of the hysterons along the field axis, there is no possibility to explain the CFD structure within this concept. In each magnetic state, in our Ising-type model, we can calculate the actual distribution of the interaction fields. In the center of each wire, all the other wires are creating a field which is the interaction field. In one state, the interaction fields are characterized by a histogram that approximates a continuous interaction field distribution for very large systems.

In Fig. 3, we present histograms of the interaction fields in several states on the descending branch of the MHL. We see a remarkable change in the histograms as a function of the total magnetic moment. In saturated states, interaction fields have a large dispersion that can be explained as boundary effect. Nanowires from the center of the array are subjected to the largest interaction field, while peripheral nanowires felt the smallest interaction field, the difference between them being of about 150 Oe for parameters presented in Sec. III. The boundary effect cannot be neglected in arrays of ferromagnetic nanowires, mainly due to the long range interaction fields. If the analysis would be performed only for the central part of a large array, the distribution of interaction fields would have a smaller dispersion. In larger systems, the interaction field distributions in all the magnetic states have a decreased dispersion but the minimum dispersion is always found in demagnetized state. During the magnetization reversal on the descending MHL, the interaction field distribution moves, and in the same time its variance changes drastically becoming very small in demagnetized state, as observed in Fig. 3. This can be explained as a result of long range magnetostatic interactions in zero total moment state. Essentially, we can observe that in the expression of the interaction field, given by Eq. (5), if $x$ is much smaller than $L$, practically the interaction field does not depend, in a critical manner, on the distances between wires. The most important element in the evaluation of the interaction field becomes the total magnetic moment of the sample. If the sample is demagnetized, $m = 0$, we see in Fig. 3 that the corresponding histogram is very narrow around zero interaction field. This fact is strictly correct only for networks of long wires and becomes less accurate for similar systems with smaller length, like the patterned media. For long wires, the interaction field distribution has a minimum dispersion in the demagnetized state and for short wires it has a maximum dispersion in that state. This distinction between the two types of arrays is reflected in specific FORC diagram fingerprint, patterned media having the “wishbone” structure diagram.

To clarify the relation between the FORC diagram and the Preisach distribution defined with intrinsic coercivities
and state dependent statistical interactions, we represent in the same figure (Fig. 4) the FORC distribution obtained from an “experimental” FORC set calculated in the 0 K-IPM and Preisach distributions in five states on the descending branch of the MHL (the ones presented in Fig. 3). In each state, we have calculated the interaction field for every wire in the Ising-type model. The intrinsic coercivity and the calculated interaction field are the coordinates of the points representing the Preisach hysteresis in the given state. As the intrinsic coercivity is not state dependent, we clearly see how the Preisach distribution is moving only along the interactions field axis.

In Fig. 4, we denoted with $H_A$ and $H_B$ the extremes of the IFD extension along the interaction field axis, observing that the coercivity in B is roughly the mean of the intrinsic distribution, $H_{C0} = 150$ Oe, and $H_A$ is the maximum interaction field in the saturated state, characteristic to the nanowire array center. One observes that the IFD appears tilted with a small angle from the interaction field axis, which is consistent with many experimental diagrams, except for the samples with very narrow distributions of coercive fields. $\Delta H_{CFD}$ is the CFD extension measured from the point $A_0(H_{C0},0)$, noting also that the center of the IFD do not correspond to the point $A_0$. One sees that the five distributions are not perfectly superimposed over the IFD.

A partial conclusion concerning this result is that the IFD feature from the FORC distribution does not correspond actually with the Preisach distribution in a given state. The Preisach distribution, as defined in this section, is moving along the IFD and we see that the vertical limits of the IFD are actually giving the real limits of the interaction fields when the system is going from positive to negative saturation along the MHL. The IFD component of the FORC diagram looks like a static image of a dynamic process (similar to a stroboscopic photo).

V. SWITCHING FIELD BASED PREISACH MODEL

However, even if the Preisach model in the version defined in Sec. IV could give a reasonable explanation for the shape and extension of the IFD feature, it cannot give any base for understanding the CFD structure. The CFD covers, both in experimental FORC diagrams and in those modeled by us with 0 K-IPM, a wide range of coercive fields, which approximately comply to the relation $H_B \approx \Delta H_{CFD}$, as we observed in Fig. 4. The CFD shows that the interactions between the wires have a more complex effect than the one described in Sec. IV. Interactions should influence the effective coercivity of the wires, as well.

The first idea is to use the 0 K-IPM and to identify for each hysteron the actual switching fields in order to see if in reality the coercivity is influenced by the interactions. The switching fields will be used as Preisach coordinates. In the 0 K-IPM, the switching fields of the wires are calculated in the following manner: (i) on the descending branch of the major loop one identifies when a wire is switched “down” and this field is recorded as negative switching field given by: $H_{ak} = H_{app}$. On the FORCs we observe at which field the wire is switched back and one records again the positive switching field for this wire, given by: $H_{sk} = H_{app}$. For accuracy, we have used a field step of 5 Oe for each FORC, and each step divided into $10^6$ sub-intervals for switching fields’ acquisition, so in each small field step there is at most one switching event.

What is exceptionally important to say is that we have observed that for a given switch “down” of one wire, the switch “up” is different on different FORCs. This means that indeed the wire coercivity is changing in the magnetization process, which is an effect we tried to evidence in order to explain the CFD feature. The dependence of actual coercivity on the magnetization process, however, shows the complexity of this physical phenomenon even in a rather simple magnetic system like the magnetic nanowire array.

The switching field concept applied to the nanowire system opens a new set of questions to be addressed in order to understand finally the shape of the FORC diagram. One aspect is that we can define, using the switching fields, a position of the hysteron’s magnetic moment in the Preisach plane. This position is process dependent so the hysteron will move in the plane depending on the actual FORC on which we measure the switching field. This movement is similar to the shift of the Preisach distribution described in Sec. IV, when the position in the plane was calculated using the coercive and interaction fields. What is different in this case is that the movement does not preserve the intrinsic coercivity unchanged allowing in this way an explanation for the CFD FORC feature.

To make this study of switching systematic, it is important to establish the sequence of switchings along the FORCs and their relation with the FORC distribution. For simplicity, we go back to the Preisach model in the modified version called moving Preisach model to observe the sequence of switchings in very simple cases. As the nanowire system is characterized by demagnetizing type mean field interactions, we consider only the case of a MPM with a negative moving constant ($\alpha < 0$). We analyze the effect of mean field interactions on the sequence of switchings for: (i) a singular distribution along the interaction field axis (SDI); and (ii) a singular distribution along the coercivity axis (SDC). In each case, we start from the evaluation of the FORC diagram and switching events in the classical Preisach model and, after that, we evaluate the effect of the demagnetizing mean field. In Figs. 5–7, we show the results for the SDI case and in Figs. 8–10 for the SDC case.

A. SDI case

For the SDI case in CPM, let us consider $n$ groups of particles with the interaction fields in the sequence $H_{a1} > H_{a2} > \cdots > H_{an}$, where $H_{a1} = H_{u}^{\text{max}} = 150$ Oe and $H_{an} = H_{u}^{\text{min}} = -150$ Oe, with FORCs and FORC diagram presented in Fig. 5.

Using the Preisach model, it is simple to establish the order in which particles switch on the major loop (from “up” to “down” on the descending branch) and along the corresponding FORCs (from “down” to “up”). We see in Fig. 5 the sequence of switchings “down” $i_1, i_2, i_3, \ldots, i_{n-1}, i_n$ on the descending branch of the MHL, and the corresponding switchings “up” $i^\prime_1, i^\prime_2, i^\prime_3, \ldots, i^\prime_{n-1}, i^\prime_n$ on the ascending branch of the MHL. For each FORC, the particle that switches
“down” just before the FORC starts, will switch “up” first at the same magnetic moment of the sample.

A negative mean interaction field does not change the sequence of switchings, but modifies the MHL, as we present in Fig. 6 for the same SDI, considering $a = -150\text{ Oe}$. The FORC diagram presented in Figs. 6 and 7 is extended by comparison with that for the CPM system from Fig. 5, accurately highlighting the entire range of interaction fields. Computed switchings are also uniformly distributed in the same range between $H^\text{max} - |x|$ and $H^\text{max} + |x|$, as we present in Fig. 7.

Concerning the numerical algorithm used to evaluate the mixed derivative given by Eq. (2), we observe that it is based on the estimation of the change in the magnetic susceptibility on successive FORCs [see Fig. 1(a)]. In the SDI case, this change occurs only when the FORCs reach the ascending branch of the MHL.

### B. SDC case

When the singular distribution is along the coercivity axis (SDC), we have considered in the classical Preisach model $n$

non-interacting groups of particles, with coercive fields uniformly distributed in the sequence $H_{c1} < H_{c2} < \cdots < H_{cn}$, where $H_{c1} = H^\text{min} = 150\text{ Oe}$ and $H_{cn} = H^\text{max} = 250\text{ Oe}$. For this system, FORCs and the FORC diagram are presented in Fig. 8.

We observe that even if the shape of the major loops is identical with the one presented in the SDI case, the FORCs are different. As we see in Fig. 8, on each FORC, the particle that switches first is steadily the same, namely, the particle that switches “down” in $c_1$ because it has the smallest coercive field, $H_{c1}$. To identify the correspondent switchings “up,” we have used the second superscript index that accounts for the FORC. For example, on the 3rd FORC, particles switch from “down” to “up” in the sequence: $c_1^{1,3}, c_2^{2,3}, c_3^{3,3} \equiv c^3$. The sequence of switchings is the same on both branches of the MHL, denoted in Fig. 8 with $c_1, c_2, c_3, \ldots, c_{n-1}, c_n$ on the descending branch, and with $c_1^{1,n}, c_2^{2,n}, c_3^{3,n}, \ldots, c_{n-1,n}, c_n^{n,n} \equiv c^n$ on the ascending branch. Unlike the case of the singular distribution of interaction fields, for each FORC, the particle that switches “down” just before the FORC starts, will switch “up”
the last, when \( m = +1 \). Switching events are presented in Fig. 9 for the same system but with a negative mean interaction field, considering \( a = -150 \text{ Oe} \). The sequence of switchings remains unchanged but the switching fields become asymmetrical, leading to an asymmetrical FORC diagram (with respect to the coercivity axis) with the wishbone shape presented in Figs. 9 and 10.

This analysis evidences a rather unexpected fact. We see in Fig. 10 that the most prominent feature of the diagram (branch AB) is in fact not related to switches of different wires. The AB branch is the effect of the switches of the same wires (the smallest coercive wires) in different environments characteristic to different FORCs. Real switchings of the other wires are evidenced on the AC branch. Each segment between the two main branches, shown in Fig. 10, can be associated to a FORC. FORC diagram algorithm is providing information only concerning the slope changes of the consecutive FORCs, and as a result one obtains the “shadow” branch AB and the “switching” branch AC, as we see in Fig. 10. The coordinates of the three points in the \((H_c, H_u)\) plane are: \( A(H_c^\text{min}, |x|) \), \( B(H_c^\text{MHL}, -H_c^\text{MHL} + H_c^\text{min} - |x|) \), and \( C(H_c^\text{max} + |x|, 0) \), where \( H_c^\text{MHL} \) is the average of the intrinsic coercive fields.

Another effect of the negative interaction mean field related to the coercive field is easily observed comparing the FORC diagrams presented in Figs. 8 and 9. While the intrinsic coercive field distribution is in a range between \( H_c^\text{min} \) and \( H_c^\text{max} \), a negative mean interaction field leads to an extension of the coercivities between \( H_c^\text{min} \) and \( H_c^\text{max} + |x| \), also observed in the FORC diagram from Fig. 10 on the AC branch.

In summary, in a Preisach-type model (MPM) mean field interactions could be responsible for larger apparent coercive fields. The switching events are consistent in the SDC case with the “reversal field memory” effect.\(^{33}\)

C. Physical model for nanowire arrays (0 K-IPM)

In this section, we present the algorithm used to provide a Preisach-type image based on switching fields. As it has been shown in Secs. VA and VB, the switching fields are dependent on the magnetic state of the sample. In the case of the FORC measurement starting on the descending branch of the MHL, one of the two switching fields (the negative switching field—from “up” to “down”) is uniquely established. The positive switching field is “FORC dependent,” that is, on each FORC, the same wire will have a different positive switching field. To provide a geometrical image of these switches we propose to take as positive switching value the field measured on the FORC starting in the negative switching field of the wire. In this way, we define the Preisach coordinates of each hysteron representing a wire. As we did for the coercivity/interaction field image in Fig. 4, we superimpose in Fig. 11(a) the same FORC diagram (calculated in the 0 K-IPM for \( a = 250 \text{ nm} \)) with the points
represented in the positions given by the switching fields of the hysterons.

One observes in Fig. 11(a) that the density of the hysterons represented in the explained manner is not entirely covering the FORC diagram. Nonetheless, the points are following rather exactly the \( \text{AA}_0 \) and \( \text{A}_0\text{C} \) lines. Contrary to the shape of the FORC diagram, the concentration of hysterons is higher along the \( \text{A}_0\text{C} \) line than along the \( \text{AA}_0 \) line. We see that the small coercivity wires are found along the \( \text{AA}_0 \) line. The coercive fields measured on the FORC diagram are approximately equal to small intrinsic coercivities of the wires. The wires located in the center of the network are represented near the point A (strongest interactions) while the wires near the border are represented at smaller interactions. The sequence of switchings observed in the SDI case (see Fig. 7) is rather well respected.
Nanowires with larger intrinsic coercive fields are represented in the Preisach plane, in Fig. 11(a), along the $A_0C$ line and they almost exactly cover the CFD feature. It should be mentioned however that the coercivities observed on the FORC diagram are much larger than the intrinsic ones. The largest intrinsic coercivity in the array was 219 Oe and both on the FORC diagram and in the switching field’s representation we got more than 400 Oe (almost the double value), as we observe in Fig. 11(a). It can be shown that in fact the maximum coercivity is the sum between the largest intrinsic coercivity and the demagnetizing field at saturation in the center of the sample (see also the coordinates of the point C in Figs. 9 and 10).

The strange behavior described in the previous paragraphs can be understood rather easily. The main fact to be considered is that in nanowire systems the interactions are essentially mean field interactions of demagnetizing type. The switches represented along the $A_0C$ line are produced on FORCs starting on positive magnetic moments of the sample. The mean field has the same direction from the starting point on the FORC up to the positive saturation. The loops of the hysterons (representing lower coercivity wires) will be shifted along the interaction field axis. For the FORCs starting on the negative values of the magnetic moment the behavior is different. The switch down is reached when the demagnetizing field is on the positive direction (at $m < 0$) and the switch up is executed when the direction of the demagnetizing field has already changed (at $m > 0$). As a result, the apparent coercivity is larger and the hysteron has an almost symmetrical loop.\(^{35}\)

As an example, switching events for two selected nanowires are highlighted in Fig. 11(b) analyzing, as time dependence (field steps), the applied field and the effective field on the descending branch of the MHL until reversal fields are reached, and on the FORCs that start from the reversal fields. When the effective field becomes $-H_{1,2}$ on the MHL, the selected nanowires switch “down,” and when the effective field reaches $+H_{1,2}$ value on the corresponding FORC, the nanowires switch back “up.” Following dashed arrows from Fig. 11(b), switching fields, $H_s$ and $H_\beta$, are easily identified. Using Eqs. (2) and (3), we have calculated Preisach hysterons’ coordinates $P_{1,2}(H_c, H_h)$ to identify both switching events in Fig. 11(a).

$P_1$ is obtained for the nanowire with the intrinsic coercive field $H_{1,1} = 126.9$ Oe, which switches “down” just before the FORC\(_1\) starts at $m = 0.84$, and switches “up” on the FORC\(_1\). We found for this nanowire a rather small difference between the intrinsic coercive field and the one obtained from switching fields (138.8 Oe). $P_2$ is obtained for the nanowire with the intrinsic coercive field $H_{1,2} = 177.7$ Oe, which switches “down” just before the FORC\(_2\) starts at $m = -0.93$, and switches “up” on the FORC\(_2\). For this nanowire, the point $P_2$ has as coordinates the coercive field (378.7 Oe) much larger than the intrinsic one, and a small interaction field (10.2 Oe), the corresponding Preisach hysterons being almost symmetrical. So, the first nanowire roughly keeps their coercivity, while the second has a larger apparent coercive field, due to the interactions, that give rise to the “reversal field memory” effect observed in nanowire arrays.\(^{14}\)

VI. CONCLUSION

This broad analysis is offering the tools for understanding the shape of the FORC distribution/diagram and to provide the right method to exploit the experimental data.

The relation between the hysterons and the FORC diagram is presented in Fig. 12. The IFD feature is related with the FORC dependent switchings of the nanowires with small intrinsic coercive fields. In the figure, we show the switching points of the smallest coercivity wire (with circles) on successive FORCs. The switching of the same wire will be accounted again and again within the AB limits of the IFD. The significance of this feature is similar to the one discussed for the “shadow” line AB in Fig. 10.

The nanowire with the largest coercive field, with switching points represented with rectangles in Fig. 12, is present in the FORC diagram in the point C. IFD and CFD features can be understood if we use both Preisach-type representations of the hysterons in the Preisach plane. The intrinsic coercivity/interactions image gives the understanding for the IFD feature and the switching type image is providing the insight for the CFD region.

The five distributions presented in Fig. 4 are not perfectly overlapped over the IFD, but allow us identifying a
represents the mean of the distribution, \( H_{c0} = 150 \text{ Oe} \) (in the same way in which in Fig. 10, the coercivity of the point B, \( H_{cMHL} \), represents the mean of the uniform distribution of coercive fields). We observe in Fig. 11(a) that the first switchings on the descending MHL are of the smallest coercive wires, emphasized in the vicinity of the point A, at the smallest values of the coercive field in the FORC diagram, denoted \( H_{c_{\min}} \). The standard deviation will be estimated by \( H_{c} = (H_{c0} - H_{c_{\min}}) / 3 \). Nanowires with larger coercive fields are strongly influenced by interactions, switching events being overlapped on the CFD feature.

This study evidences a number of apparently contradictory characteristics of the FORC diagram for nanowire arrays:

(i) the distribution of intrinsic coercivities is found from the IFD component and surprisingly not from the CFD component, as presented above.

(ii) the CFD is due to real switchings but the coercivities along this feature are not indicating intrinsic coercivities of the wires within the array.

(iii) even if the IFD is not representing real switchings, it offers a good evaluation of the interactions in the system during the magnetization processes. However, IFD is giving a static image of a dynamic process in which interactions are state dependent.

Consequently, the FORC distribution is really very far from the Preisach distribution as known from the classical Preisach model. Nonetheless, when the FORC structure is profoundly understood, one can find a way to exploit correctly the information it contains. This analysis shows how dangerous could be the use of the diagram “as such” and the belief that it represents a slightly distorted distribution of the Preisach function associated to the sample. This type of systematic study should be extended for other structured magnetic systems, for example for BPM samples, in order to profoundly understand the reality behind the experimental FORC diagrams.

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